1) A particle moves along a horizontal line so that its position at any time $t \geq 0$ is given by the function $x(t) = t^2 - 4t + 3$, where x is measured in feet and t is measured in seconds.

(a) Find the displacement of the particle during the first 3 seconds. Explain its meaning.

$x(3) - x(0) = -3$

At $t=3$, the particle is 3 units to the left of its initial position.

(b) Find the average velocity of the particle during the first 3 seconds. Explain its meaning.

$\frac{x(3)-x(0)}{3-0} = -1$ ft/sec From $t=0$ to $t=3$, the particle had an average velocity of -1 ft/sec.

(c) Find the particle's initial velocity and its velocity at $t=3$ seconds. Explain the meanings of each in terms of the particle's movement.

$v(t) = 2t - 4$ $v(0) = -4$ At $t=0$, the particle is moving backward at 4 ft/sec

$v(3) = 2$ ft/sec At $t=3$, the particle is moving forward at 2 ft/sec

(d) Find the acceleration of the particle when $t=3$ seconds. Explain its meaning in terms of the particle's velocity.

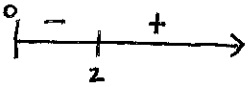
$a(t) = 2$ ft/sec^2. The velocity of the particle is increasing.

(e) At $t=3$ seconds, is the speed of the particle increasing or decreasing? Justify.

$v(3) > 0$ The speed of the particle is increasing since
$a(3) > 0$ $v(3)$ & $a(3)$ have the same sign.

(f) During what times is the particle moving to the right? Left? At what values of t does the particle change direction? Justify.

$v(t) = 0$
$t = 2$



The particle moves left from $t=0$ to $t=2$ b/c $v(t) < 0$. The particle moves right on $(2, \infty)$ b/c $v(t) > 0$. At $t=2$, the particle changes direction b/c $v(t)$ changes signs.

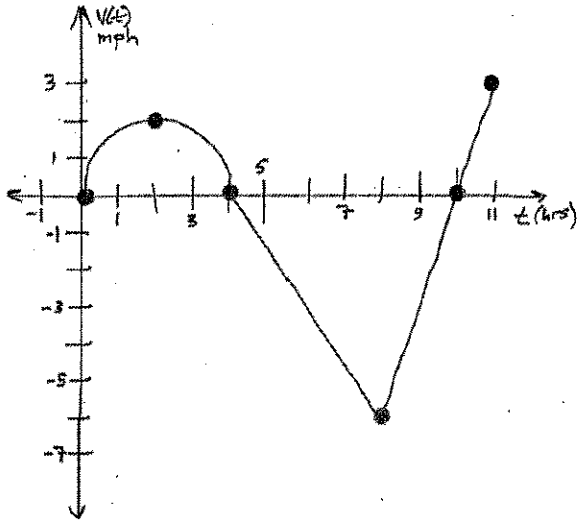
(g) Find the total distance the particle travels during the first 3 seconds. Are you as exhausted as the particle?

$|x(2) - x(0)| + |x(3) - x(2)| = |-1 - 3| + |0 - (-1)|$

$= 4 + 1$

$= 5$

2)



The graph above shows the velocity, $v(t)$, in miles per hour of a particle moving along the x-axis for $0 \le t \le 11$ hours. It consists of a semi circle and two line segments. Use the graph and your knowledge of motion to answer the following questions.

(a) At what time, $0 \le t \le 11$ hours, is the speed of the particle the greatest?

The speed of the particle is the greatest at $t = 8$.

(b) At which of the times, $t = 2$, $t = 6$, or $t = 9$ hours, is the acceleration of the particle greatest? Justify.

$a(2) = 0$

$a(6) < 0$

$a(9) > 0$

At $t = 9$, the acceleration is greater than @ $t = 2$ & $t = 6$.

(c) Over what open time interval(s) $0 < t < 11$ hours is the particle moving to the left? Justify.

The particle is moving left on $[4, 10]$ b/c $v(t) < 0$.

(d) Over what open time interval(s) $0 < t < 11$ hours is the velocity of the particle increasing? Justify.

The velocity is increasing $(a > 0)$ on $(0, 2)$ & $(8, 11)$

(e) Over what open time interval(s) $0 < t < 11$ hours is the speed of the particle increasing? Justify.

The speed of the particle is increasing on $(0, 2)$, $(4, 8)$, & $(10, 11)$ b/c $v(t)$ & $a(t)$ have the same sign.

(f) At what times on $0 < t < 11$ is the acceleration of the particle undefined?

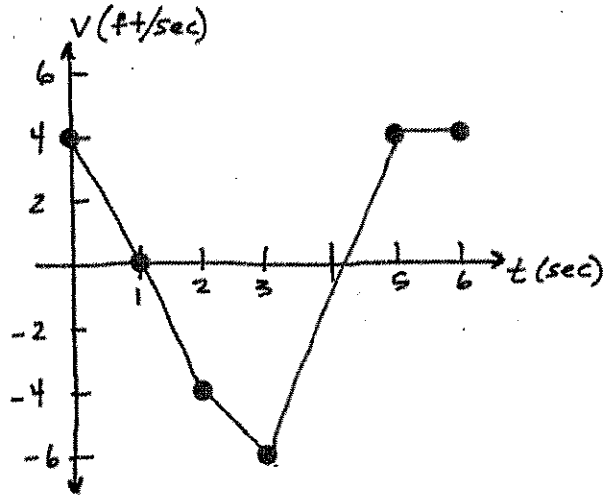
$a(t) = \emptyset$ @ $t = 4$.

(g) Find the area of the semicircle on the interval $0 \le t \le 4$ bounded by the curve and the x-axis, then find the area of the triangle on the interval $4 \le t \le 10$ bounded by the curve and the x-axis, and finally, find the area of the triangle on the interval $10 \le t \le 11$ bounded by the curve and the x-axis. If all of these areas were positive and added together, propose what quantity this might be in terms of the particle's movement on $0 \le t \le 11$ hours.

Total Distance Travelled by the particle, in miles, from $t = 0$ to $t = 11$ hrs. $= \int_0^{11} |v(t)|\,dt = \frac{1}{2}\pi(2)^2 + \frac{1}{2}(6)(8) + \frac{1}{2}(1)(3)$

$= 2\pi + \frac{51}{2}$

3)



The graph above shows the velocity $v(t)$ of a particle, in ft/sec, moving along a horizontal line for $0 \le t \le 6$ seconds.

(a) On what open intervals or at what time(s) $0 < t < 6$ is the particle at rest? Justify.

The particle is at rest @ $t = 1$ & $t = 4.2$ b/c $v(t) = 0$

(b) On what open intervals $0 < t < 6$ is the particle moving to the right? Justify.

The particle is moving to the right on $[0,1)$ & $(4.2, 6]$ b/c $v(t) > 0$.

(c) On what open intervals or at what time(s) $0 < t < 6$ is the particle moving at its greatest speed? Greatest velocity?

The greatest speed is achieved at $t = 3$.

The greatest velocity is achieved at $t = 0$ & on $[5,6]$

(d) On what open intervals or at what time(s) $0 < t < 6$ is the particle's speed increasing? Decreasing? Justify.

The speed of the particle is increasing on $(1,2)$ $(2,3)$ & $(4.2, 5)$ b/c $v(t)$ & $a(t)$ have the same sign.

The speed is decreasing on $(0,1)$ & $(3, 4.2)$ b/c v & a have different signs.

(e) What is the particle's acceleration at $t = 4.8$ second? Explain what this number means in terms of the particle's velocity.

$a(4.8) = 5$ ft/sec^2. The velocity of the particle is increasing.

(f) On what open intervals or at what time(s) $0 < t < 6$ is the acceleration of the particle the greatest?

On $(3,5)$, the acceleration is the greatest

(g) (is for "genius") What is the particle's displacement during the 2 seconds? Justify.

$$\int_0^2 v(t)\,dt = 0$$